Technical Comments

Comment on "Stability of Multidimensional Linear Time-Varying Systems"

Peter C. Müller*
University of Wuppertal
Wuppertal, Federal Republic of Germany

IN the paper[1] by Shrivastava and Pradeep[1] seven theorems on stability of multidimensional linear second-order systems with time-varying coefficients have been discussed. Unfortunately, all statements on asymptotic stability are useless because they are based on contradictory conditions. For example, theorem 1 is considered where, within the "sufficient" conditions for asymptotic stability, a positive definite mass matrix and a negative definite time derivative of the mass matrix are simultaneously required:

$$M(t) > 0, \qquad M'(t) < 0 \tag{1}$$

With respect to the definitions of positive or negative time-variant functions,[2] the requirements (1) imply the existence of constant positive definite matrices M_1 and M_2 such that

$$M(t) \geq M_1 > 0, \qquad M'(t) \leq -M_2 < 0 \tag{2}$$

for all $t \geq t_0$. Therefore we have

$$M(t) = M(t_0) + \int_{t_0}^{t} M'(\tau)\mathrm{d}\tau \leq M(t_0) - M_2(t-t_0) \tag{3}$$

At least for

$$t > t_0 + \frac{\lambda_{\max}[M(t_0)]}{\lambda_{\min}(M_2)} \tag{4}$$

inequality (3) leads to a contradiction of $M(t) > 0$.

Such contradictory conditions [Eq. (1)] also appear in theorems 1-5 and corollaries 1.1 and 3.1. Therefore, all results on asymptotic stability cannot be used.

If the asymptotic stability of mechanical systems has to be shown by the suggested Liapunov functions V, then the results of Krasovskii[2] have to be applied, showing that each nontrivial half-trajectory does not satisfy identically $\dot{V} \equiv 0$. In the case of linear systems, this analysis can be replaced by certain observability or controllability conditions. For time-invariant mechanical systems particularly, this discussion leads to the notion of pervasive damping ensuring asymptotic stability.[3]

A last comment corresponds to the stability part of theorems 4 and 5. The well-known lemma leads to the very conservative conditions (25) and (27). For example, condition (27) requires the existence of a matrix L with

$$A = K'L \tag{5}$$

i.e., the condition

$$\text{rank}\, K' = \text{rank}[K'A] \tag{6}$$

is necessary for the possibility that Eq. (27) may hold. In application, this rank condition is very rarely satisfied for systems with many degrees of freedom. In particular, the theorems cannot be applied to time-invariant circulatory systems. Further results on this subject are reported by the author.[3]

References

[1]Shrivastava, S.K. and Pradeep, S., "Stability of Multidimensional Linear Time-Varying Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 8, Sept.-Oct. 1985, pp. 579-583.

[2]Krasovskii, N.N., *Stability of Motion,* Stanford University Press, Stanford, CA, 1963, pp. 8, 66-68.

[3]Müller, P.C., *Stabilität und Matrizen,* Springer-Verlag, Berlin, 1977.

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*Professor, Department of Safety Control Engineering.

Reply by Authors to P.C. Müller

Shashi K. Shrivastava* and S. Pradeep†
Indian Institute of Science, Bangalore, India

WE wish to thank Professor Müller for his interest in our paper.[1] However, we do not agree with his comment that "M' being negative definite is inconsistent with M being positive definite." Here is a counterexample:

$$M = \begin{bmatrix} 4e^{-4t} & 0 \\ 0 & 2e^{-2t} \end{bmatrix} > 0$$

$$M' = \begin{bmatrix} -16e^{-4t} & 0 \\ 0 & -4e^{-2t} \end{bmatrix} < 0 \qquad \text{for all } t \in [0,\infty)$$

One can find many such examples. Therefore the "sufficient" conditions for asymptotic stability in theorems 1-5 and corollaries 1.1 and 3.1 are applicable in a number of situations.

The results of Krasovskii[2] cannot be applied to the system under consideration. The analysis in Ref. 2 is restricted to equations with periodic/constant coefficients, for which a number of well-known theorems exist, whereas the theorems of Ref. 1 deal with arbitrarily time-varying systems.

With regard to the last comment, the authors fail to understand the significance of condition (5) of Müller. Equation (27) of Ref. 1 is

$$\left[\begin{array}{c|c} -K' & -A \\ \hline A & 2D-M' \end{array}\right] \geq 0 \qquad \text{for all } t \in [0,\infty) \tag{3}$$

It is not clear why this requires the existence of a matrix L with

$$A = K'L \tag{2}$$

If K' is nonsingular (as is the case in most time-varying systems), then, trivially, L exists and $L = (K')^{-1}A$ (although it is not clear what the role of L is). If K is singular, then L does not exist. However, this does not prevent Eq. (1) from

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*Professor. Associate Fellow AIAA.

†J.R.D. Tata Research Fellow.

being satisfied. For instance, consider time-invariant circulatory systems where $M' = K' = D = 0$; Eq. (1) becomes

$$\left[\begin{array}{c|c} 0 & -A \\ \hline A & 0 \end{array}\right] \geq 0 \qquad \text{for all } t\epsilon[0,\infty) \tag{1}$$

which is trivially satisfied, although L does not exist, owing to the singularity of K'.

References

[1]Shrivastava, S.K. and Pradeep, S., "Stability of Multidimensional Linear Time-Varying Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 8, Sept.-Oct. 1985, pp. 579–583.

[2]Krasovskii, N.N., *Stability of Motion,* Stanford University Press, Stanford, CA, 1963, pp. 66–68.

Book Announcements

ASHWORTH, M.J., Royal Naval Engineering College, ***Feedback Design of Systems with Significant Uncertainty,*** John Wiley and Sons, New York, 1982, 246 pages. $40.

Purpose: Feedback control theory is developed allowing for the fact that the mathematical model of the hardware to be controlled is only a theoretical representation of a plant which will manifest all the variations of the manufacturing process and which will change according to age, environment, and its operational mode throughout its life. The analysis is limited to single-input, single-output systems.

Contents: The feedback principle. Considerations of dynamic accuracy. Low order modelling. Plant variation and system sensitivity. Loop synthesis and sensitivity reduction. Noise disturbance. Constraining the control effort. Appendices. References. Index.

BARNETT, S., University of Bradford, ***Polynomials and Linear Control Systems,*** Marcel Dekker, New York, 1983, 452 pages. $55.

Purpose: The aims of this book are twofold: first, to develop properties of polynomials and polynomial matrices using matrix techniques and second, to describe related topics in the theory of linear control systems. The book is intended both for study at the senior or graduate level and for reference. Prerequisites include matrix algebra and transform methods for differential or difference equations.

Contents: Polynomials: approaches to the greatest common divisor. Basic properties of control systems. Root location and stability. Feedback, realization, and polynomial matrices. Generalized polynomials and polynomial matrices. Appendices. Index.

RUYMGAART, P.A., Univeristy of Technology—Delft, and **SOONG, T.T.,** State University of New York at Buffalo, ***Mathematics of Kalman-Bucy Filtering,*** Springer-Verlag, New York, 1985, 168 pages. $29.

Purpose: This book presents a mathematical approach to continuous-time Kalman-Bucy filtering and is an outgrowth of lectures given by the authors. The mathematics consists mainly of operations in Hilbert spaces. Hence, Ito calculus, theory of Martingales, Markov processes, and infinite-dimensional innovations have been omitted.

Contents: Elements of probability theory. Calculus in mean square. The stochastic dynamic system. The Kalman-Bucy filter. A theorem by Lipster and Shiryayev. Appendix: solutions to selected exercises. References. Subject index.

CHEN, C.H., Southeastern Massachusetts University, ***Nonlinear Maximum Entropy Spectral Analysis Methods for Signal Recognition,*** John Wiley and Sons, New York, 1982, 170 pages. $52.95.

Purpose: This book is a research monograph based primarily on extensions of Burg's maximum entropy spectral analysis, with special emphasis on a non-linear error minimization procedure due to Fougere. Major emphasis is placed on computer implementation and application examples of spectral analysis methods.

Contents: Introduction. The univariate maximum entropy spectral analysis. Computer implementation. Signal classification, detection and resolution. On multichannel (multivariate) maximum entropy spectral analysis. On a two-dimensional maximum entropy spectral analysis method with application to texture analysis. A study of texture classification using spectral features. Bibliography. Appendices. Index.

TORBY, B.J., California State University at Long Beach, ***Advanced Dynamics for Engineers,*** Holt, Rinehart, and Winston, New York, 1984, 426 pages. $41.95.

Purpose: The purpose of this text has been to combine the teaching of advanced dynamics with the computer solution of algebraic and ordinary differential equations to enhance the learning of both subjects.

Contents: Introductory remarks. Particle dynamics: kinematics. Particle dynamics: kinetics. Systems of particles. Rigid body dynamics: kinematics. Rigid body motion. Rigid body dynamics. Energy methods. Theory of small oscillations. Digital computer simulation. Appendices. Index.

BALAKRISHNAN, A.V., University of California at Los Angles, ***Kalman Filtering Theory,*** Springer-Verlag, New York, 1984, 222 pages. $26.

Purpose: This book is intended for a one-quarter or one-semester course at the graduate level in engineering. The prerequisites are elementary state-space theory and elementary stochastic process theory. The text develops those aspects of Kalman filtering which can be given a firm mathematical basis.

Contents: Review of linear system theory. Review of signal theory. Statistical estimation theory. The Kalman filter. Likelihood ratios: Gaussian signals in Gaussian noise. Bibliography. Index.